Two recent things in the news that may have interest to various engineers:

1.) **The Spencer Dam Failure Report** https://damsafety.org/SpencerDamReport
from the American Society of Dam Safety Officials (ASDSO)
has been released to the public.

Super brief:
The name of the dam investigated is the "Spencer Dam."
It is located on the Niobrara river and forms part of the boundary between Nebraska and South Dakota.
The dam was a "run-of-the-river" dam (not built for flood control), main goal was to generate electricity.
Total power available: 3 MW. (compare to 500 MW for an average coal-fired generation station)
Remained in service because cheaper than decommissioning, but 3 MW is no longer significant on the grid.
The dam failed in March of 2019 during a spring flood.
The proximate cause of the failure was an ice-jam that blocked spillways and frozen gates that would not open.
Lives were lost, much damage was done downstream from the resulting flood.
The dam will most likely be demolished and the river will be allowed to return to a natural condition.

2.) **The FCC has voted to open a 6 GHz band to "ISM" unlicensed operations**. (Industrial, Scientific, Medical)
This will enable about 59 new Wi-Fi channels.
Expect to see new Wi-Fi routers, etc.
This is also a big deal for the mobile phone companies because it competes with mobile hot-spots typically served by "cellular data."
https://www.theverge.com/2020/4/23/21231623/6ghz-wifi-6e-explained-speed-availability-fcc-approval
https://www.fcc.gov/document/fcc-opens-6-ghz-band-wi-fi-and-other-unlicensed-uses

1

The two equations we have derived are called the Telegrapher's Equations

$$\frac{\partial^2 v(x,t)}{\partial x^2} = LC\frac{\partial^2 v(x,t)}{\partial^2 t} + (LG + CR)\frac{\partial v(x,t)}{\partial t} + RGv(x,t)$$

$$\frac{\partial^2 i(x,t)}{\partial x^2} = LC\frac{\partial^2 i(x,t)}{\partial^2 t} + (LG + CR)\frac{\partial i(x,t)}{\partial t} + RGi(x,t)$$

For short transmission lines such as are usually encountered in microcontroller situations we can assume that the transmission line is lossless. That is, $R = 0$ and $G = 0$.

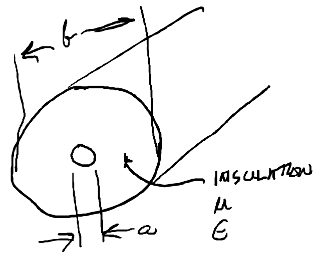
$$\left.\begin{aligned}\frac{\partial^2 v(x,t)}{\partial x^2} &= LC\frac{\partial^2 v(x,t)}{\partial^2 t}\\ \frac{\partial^2 i(x,t)}{\partial x^2} &= LC\frac{\partial^2 i(x,t)}{\partial^2 t}\end{aligned}\right\}\ \text{Wave Equations}$$

Let $v(x,t) = \cos(\omega(t - x/V_P))$ where ω = frequency (rad/sec) and V_P = propagation speed (m/s)

Then $\frac{\partial v^2(x,t)}{\partial t^2} = -\omega^2\cos\left(\omega\left(t - \frac{x}{V_P}\right)\right)$ and $\frac{\partial v^2(x,t)}{\partial x^2} = -\frac{\omega^2}{V_P}\cos\left(\omega\left(t - \frac{x}{V_P}\right)\right)$

Substituting these into the voltage equation leads to velocity of propagation, $V_P = 1/\sqrt{LC}$

2



The two equations we have are called the <u>Wave Equations—the lossless case</u>

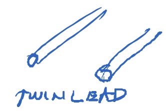
$$\frac{\partial^2 v(x,t)}{\partial x^2} = LC\frac{\partial^2 v(x,t)}{\partial^2 t}$$

$$\frac{\partial^2 i(x,t)}{\partial x^2} = LC\frac{\partial^2 i(x,t)}{\partial^2 t}$$

And we have found that $V_P = 1/\sqrt{LC}$

From the physics of a <u>coaxial cable</u> we find that $L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$ and $C = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$ where a =inner cond dia, b = shield inner dia.

Thus $V_P = \frac{1}{\sqrt{\mu\epsilon}}$



where $\mu = \mu_R\mu_0$ = the magnetic permeability of the conductor. $\mu_0 = 1.25663706 \times 10^{-6}$ m kg s^{-2} A^{-2}
And $\epsilon = \epsilon_R\epsilon_0$ = the electrostatic permittivity of the dielectric. $\epsilon_0 = 8.85418782 \times 10^{-12}$ m^{-3} kg^{-1} s^4 A^2

Note that $\frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792458 \times 10^8 = c$, the speed of light and that μ_R and ϵ_R are usually a little more than unity.

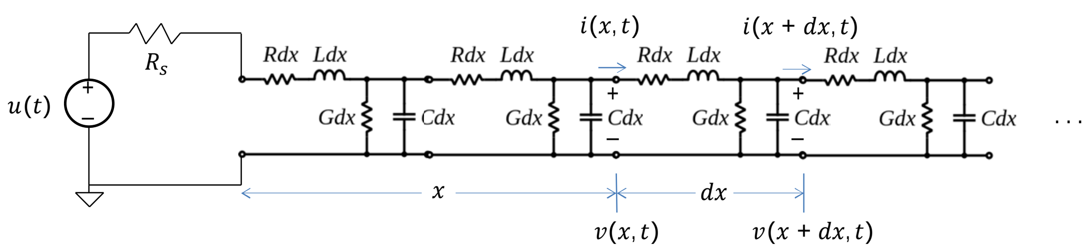
We define the velocity factor as $\frac{V_P}{c} = 1/\sqrt{\mu_R\epsilon_R}$ which is typically 60 to 80% of c.

The cable's diameter does not matter. Only $V_P/c = 1/\sqrt{\mu_R\epsilon_R}$ matters.

For other cable shapes, for example twin-lead, the math for L and C is different, but amazingly, always $V_P/c = 1/\sqrt{\mu_R\,\epsilon_R}$

3

Now, find the ratio of voltage to current (for ∞ long cable) to find the characteristic impedance of the cable.



$$v(x+dx) = v(x,t) - Ri(x,t)dx - L\frac{\partial i(x,t)}{\partial t}dx \qquad i(x+dx) = i(x,t) - Gv(x,t)dx - C\frac{\partial v(x,t)}{\partial t}dx$$

$$v(x+dx) - v(x,t) = -Ri(x,t)dx - L\frac{\partial i(x,t)}{\partial t}dx \qquad i(x+dx) - i(x,t) = -Gv(x,t)dx - C\frac{\partial v(x,t)}{\partial t}dx$$

$$\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t} \qquad \frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C\frac{\partial v(x,t)}{\partial t}$$

Lossless case: $R = 0$ Lossless case: $G = 0$

$$\frac{\partial v(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t} \qquad \frac{\partial i(x,t)}{\partial x} = -C\frac{\partial v(x,t)}{\partial t}$$

4

The two equations are: (But I only need one of them to find the characteristic impedance.)

$$\frac{\partial v(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t} \quad (1) \qquad\qquad (2) \quad \frac{\partial i(x,t)}{\partial x} = -C\frac{\partial v(x,t)}{\partial t}$$

Let $v(x,t) = \cos(\omega(t - x/V_P))$ where ω = frequency (rad/sec) and V_P = propagation speed (m/s)

$$\frac{\partial v(x,t)}{\partial x} = \frac{\omega}{V_P}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right)$$

5

The two equations are:

$$\frac{\partial v(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t} \qquad\qquad \frac{\partial i(x,t)}{\partial x} = -C\frac{\partial v(x,t)}{\partial t}$$

Let $v(x,t) = \cos(\omega(t - x/V_P))$ where ω = frequency (rad/sec) and V_P = propagation speed (m/s)

$$\frac{\partial v(x,t)}{\partial x} = \frac{\omega}{V_P}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right)$$

Substitute this into the first equation above

$$\frac{\omega}{V_P}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right) = -L\frac{\partial i(x,t)}{\partial t}$$

6

The two equations are:

$$\frac{\partial v(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t} \qquad \frac{\partial i(x,t)}{\partial x} = -C\frac{\partial v(x,t)}{\partial t}$$

Let $v(x,t) = \cos(\omega(t - x/V_P))$ where ω = frequency (rad/sec) and V_P = propagation speed (m/s)

$\frac{\partial v(x,t)}{\partial x} = \frac{\omega}{V_P}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right)$ Substitute this into the first equation above and solve for $i(x,t)$

$$\frac{\omega}{V_P}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right) = -L\frac{\partial i(x,t)}{\partial t}$$

$$\frac{\partial i(x,t)}{\partial t} = -\frac{\omega}{V_P L}\sin\left(\omega\left(t - \frac{x}{V_P}\right)\right)$$

$$i(x,t) = \frac{1}{V_P L}\cos\left(\omega\left(t - \frac{x}{V_P}\right)\right)$$

7

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$$i(x,t) = \frac{1}{V_P L}\cos\left(\omega\left(t - \frac{x}{V_P}\right)\right)$$

By definition
Impedance is the ratio of voltage to current for sinusoidal signals.

$$Z_c = \frac{v(x,t)}{i(x,t)}$$

Z_c is called the *characteristic impedance* of the transmission line.
It is a model parameter.
You can't measure it with a DMM.

8

The two equations are:

$$\frac{\partial v(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t} \qquad \frac{\partial i(x,t)}{\partial x} = -C\frac{\partial v(x,t)}{\partial t}$$

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$$Z_c = \frac{v(x,t)}{i(x,t)} = LV_P = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

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9

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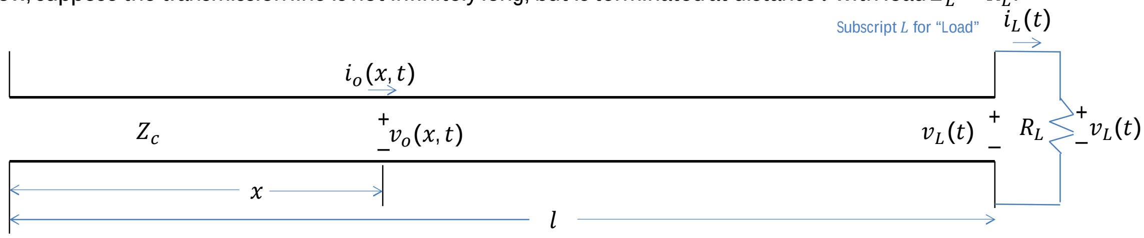
characteristic impedance
of the transmission line.
It is a model parameter.
You can't measure it with a DMM.

If there are losses Z_C becomes a function of ω, but L and C still tend to dominate. especially at higher frequencies.

$$Z_c(\omega) = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

10

Now, suppose the transmission line is not infinitely long, but is terminated at distance l with load $Z_L = R_L$.



At the load end $\frac{v_L(t)}{i_L(t)} = R_L$ and "anywhere" (at location x) we have $\frac{v_o(t)}{i_o(t)} = Z_c$ But in general, $Z_c \neq R_L$ therefore. . .

How can this be? If $x = l$ we have $Z_C = \frac{v_o(t)}{i_o(t)} = \frac{v_L(t)}{i_L(t)} = R_L$ but I'm told that in general $Z_C \neq R_L$. Contradiction???

11

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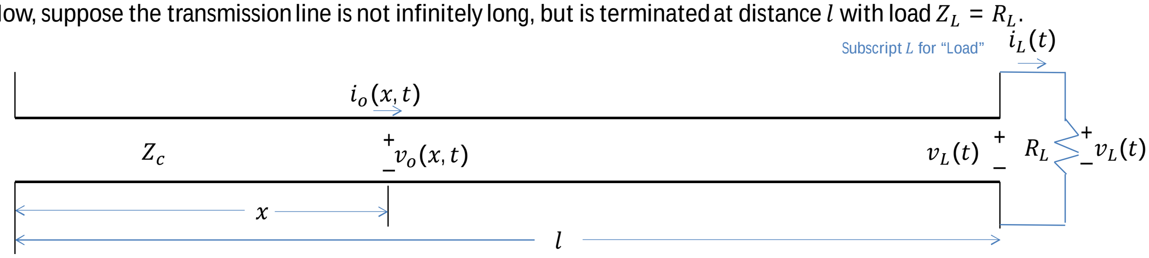

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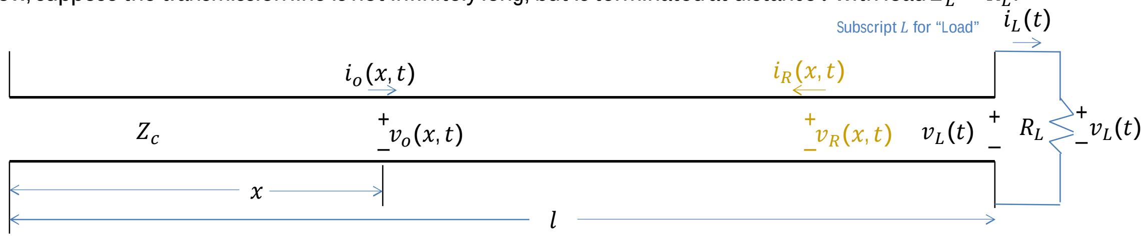
R_L is not part of the transmission line.

Suppose the transmission line is lossless. Then $Z_C \in \mathbb{R}$.
Then a comparison such as $R_L > Z_C$ are valid. (In general, cannot compare a real number to a complex number.)
Assume $R_L > Z_C$

12

Now, suppose the transmission line is not infinitely long, but is terminated at distance l with load $Z_L = R_L$.



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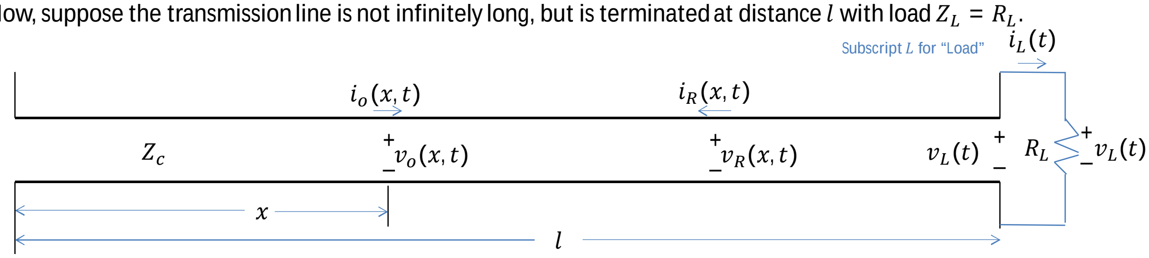
Then a comparison such as $R_L > Z_C$ are valid. (In general, cannot compare a real number to a complex number.)
Assume $R_L > Z_C$

The amount of i_o arriving at R_L will be greater than the amount of current through R_L if the voltage does not change.
Thus, the voltage will rise at the load. The load-end of the t-line will start charging its parasitic capacity.
Denote the charging current as a reflection current. Consider v_L as the superposition (sum) of v_o (incident) and v_R (reflected)
Also i_L = superposition of i_o and i_R.

(If $R_L < Z_C$ then the voltage will fall, t-line will discharge. . .)

13

Now, suppose the transmission line is not infinitely long, but is terminated at distance l with load $Z_L = R_L$.



At the load end $\frac{v_L(t)}{i_L(t)} = R_L$ and "anywhere" (at location x) we have $\frac{v_o(t)}{i_o(t)} = Z_c$ But in general, $Z_c \neq R_L$ therefore. . .

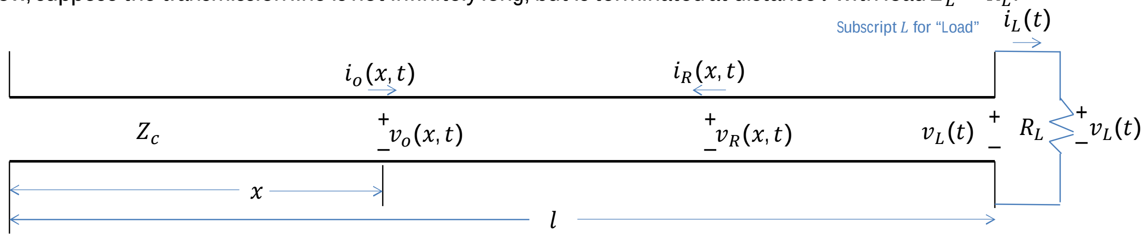
If $R_L > Z_C$ some of the incident current, $i_0(x,t)$ will reflect back and $v_L(t)$ will increase until ohm's law is satisfied for the load. $v_L(L,t) = v_o(L,t) + v_R(L,t) = i_L(t)R_L$, but by KCL $i_L(t) = i_o(L,t) - i_R(L,t)$.

$$v_o(L,t) + v_R(L,t) = \big(i_o(L,t) - i_R(L,t)\big)R_L$$

(If $R_L < Z_C$ then $v_L(t)$ down, etc.
All the equations are the same,
only some quantities turn negative.)

14

Now, suppose the transmission line is not infinitely long, but is terminated at distance l with load $Z_L = R_L$.



At the load end $\frac{v_L(t)}{i_L(t)} = R_L$ and “anywhere” (at location x) we have $\frac{v_o(t)}{i_o(t)} = Z_c$ But in general, $Z_c \neq R_L$ therefore. . .

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$$v_o(L,t) + v_R(L,t) = \big(i_o(L,t) - i_R(L,t)\big)R_L$$

Substituting for the currents, $i_o = v_o/Z_C$ etc. gives

$$v_o(L,t) + v_R(L,t) = \big(v_o(L,t) - v_R(L,t)\big)(R_L/Z_c)$$

$$v_o\left(1 - \frac{R_L}{Z_c}\right) = -v_R\left(1 + \frac{R_L}{Z_c}\right)$$

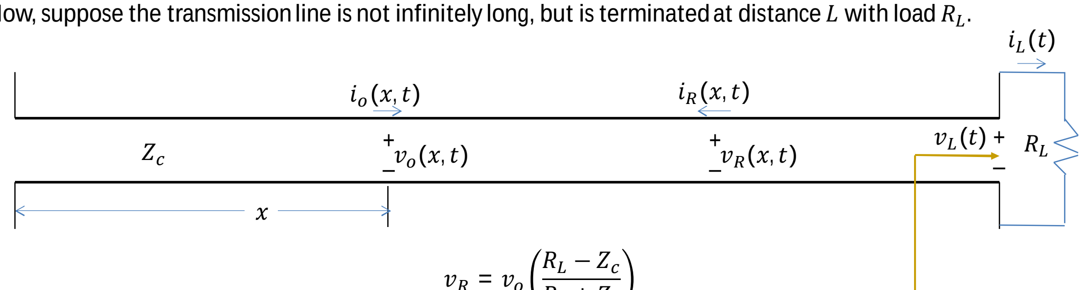
$$v_R(R_L + Z_c) = v_o(R_L - Z_c)$$

$$v_R = v_o\left(\frac{R_L - Z_c}{R_L + Z_c}\right)$$

Take this equation to the next slide.

15

Now, suppose the transmission line is not infinitely long, but is terminated at distance L with load R_L.



$$v_R = v_o\left(\frac{R_L - Z_c}{R_L + Z_c}\right)$$

Γ_L = reflection coefficient at the load

Define the reflection coefficient, $\Gamma \triangleq v_R/v_o$

Also, R_L is usually generalized to Z_L. (So in our case, $Z_L = R_L$.)

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c}$$

Then to find a reflection. . .

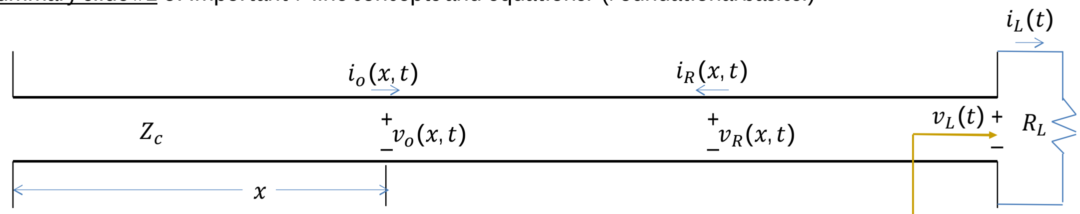
$$v_R = v_o\Gamma_L$$

Finally suppose there was an initial charge on the transmission line, defined as $v_{standing}$ and v_o represents a change in voltage traveling down the transmission line. Then by superposition

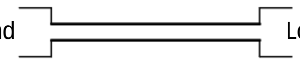
$$v_L = v_{standing} + v_{incident} + v_{reflected} = v_{standing} + v_o + v_o\Gamma$$

16

Summary slide #1 of important T-line concepts and equations. (Foundational basics.)



The schematic symbol for a transmission line is a narrowing of the lines:



Γ_L = reflection coefficient at the load

The wide ends are modeled as ordinary nodes. The narrow region represents the t-line.
A full model of the T-line requires five parameters: $L, C, R, G.$ (4 real numbers in per-length units) and length l
Equivalently, these can be represented as *characteristic impedance* Z_C, and *propagation constant* γ_C and length l
(Where in general, Z_C and γ_C are both complex numbers. (Z_C is in ohms—just ohms, not per length)

$$Z_c(\omega) = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{and} \quad \gamma_C(\omega) = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$Re\{\gamma_C(\omega)\}$ gives attenuation/length
$Im\{\gamma_C(\omega)\}$ gives phase shift/length
γ_C and Re, Im, parts are dimensionless

At any point along the line the voltage is considered as the superposition of three parts:
1.) A standing voltage that represents an initial condition, $v_{standing}$
2.) An incident wave front v_o, a change from $v_{standing}$ initiated at the source end by some action of the source.
3.) If there has been sufficient time. . . A reflection from the load, v_R. (Similarly for current.)

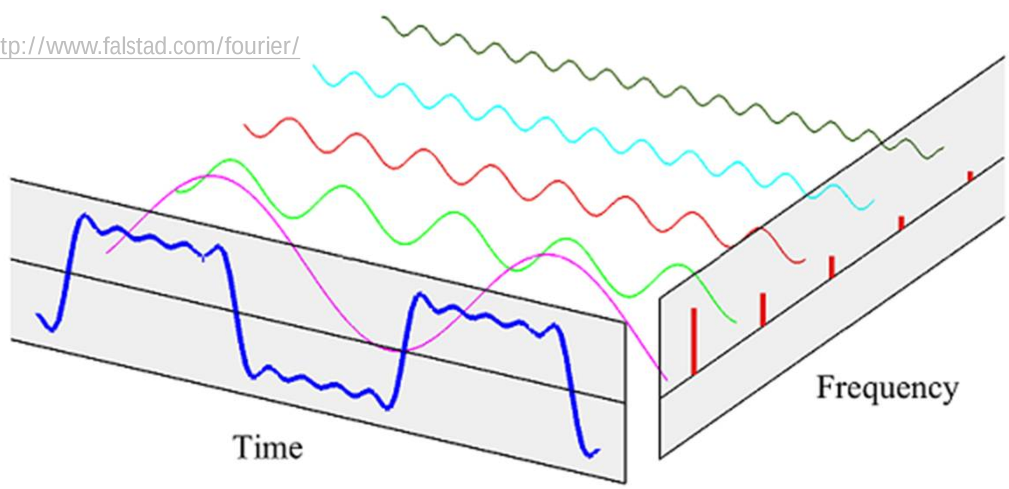
17

Fourier series of a square wave—A square wave is a sum of higher-frequency sinusoids.

See Kahn academy

https://www.khanacademy.org/science/electrical-engineering/ee-signals/ee-fourier-series/v/ee-fourier-series-intro

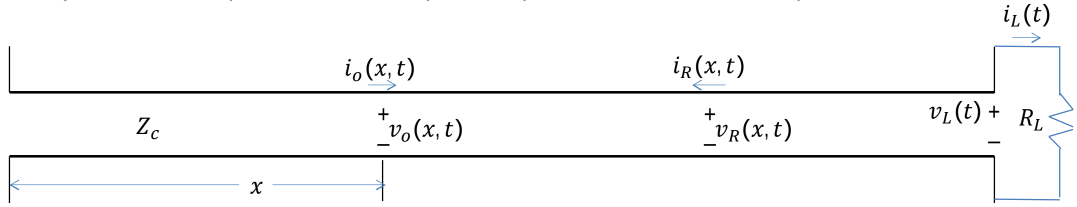
See also http://www.falstad.com/fourier/



https://wirelesspi.com/the-concept-of-frequency/

18

Summary slide #1.5 of important T-line concepts and equations. Prof. dDB can't help himself. Here comes TMI



Lossy transmission line theory involves a bunch of interesting concepts we do not have time to cover.

One of the most important things that you should hear about is that **propagation velocity is generally frequency dependent**. Typically **high-frequency waves travel more slowly**. (And with greater attenuation per unit length.)

Consequently, **information** (modulation needs BW) **travels more slowly** than the lowest-frequency components of the wave.

A modulated wave has two types of velocity: Phase velocity—speed at which a local peak moves.
Group velocity—speed of the envelope. (Typically < phase velocity)

Phase velocity is $\omega\,/\,\mathrm{Im}\{\gamma_C\}$

Group velocity is $\left(\frac{\partial \mathrm{Im}\{\gamma_C(\omega)\}}{\partial\omega}\right)^{-1} = \frac{\partial\omega}{\partial Im\{\gamma_C(\omega)\}}$

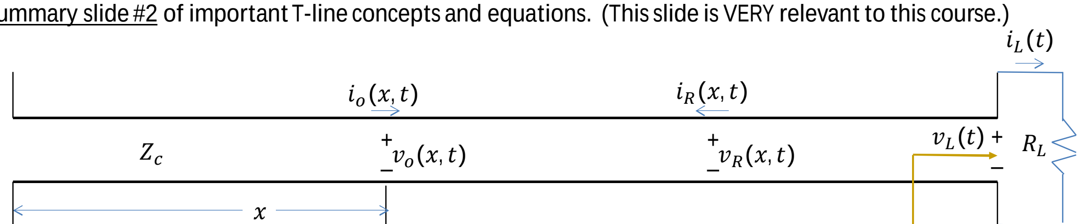
This slide is not important to this course—it is important to life!

But if lossless, none of this matters!

https://en.wikipedia.org/wiki/Group_velocity

19

Summary slide #2 of important T-line concepts and equations. (This slide is VERY relevant to this course.)



In the case of a lossless t-line (a short t-line) assuming $R = 0$ and $G = 0$ is reasonable. Then. . .

$$Z_c(\omega) = \sqrt{\frac{L}{C}} \quad \text{and} \quad \gamma_C(\omega) = j\omega\sqrt{LC}$$

Γ_L = reflection coefficient at the load

And we note that the velocity of propagation is $V_P = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\mu_R\epsilon_R}} = \frac{\omega}{|\gamma_C(\omega)|} = \frac{1}{|\gamma_C(1)|}$

We defined the reflection coefficient as the ratio of reflected voltage to incident voltage. $\Gamma = v_R\,/\,v_o$

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c}$$

By similar math, a reflection coefficient at the source can be defined.

$$\Gamma_S = \frac{Z_S - Z_c}{Z_S + Z_c}$$

20